Effects of Offsets on Bipolar Integrated Circuit Mixer Even-Order Distortion Terms

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***Abstract*—Second-order intermodulation products in bipolar double-balanced mixers can be generated due to device mismatches. These spurs are analyzed theoretically and in simulation. Guidelines are presented that show the maximum acceptable mismatch to meet a given second-order intercept-point specification.**

***Index Terms*—Bipolar analog integrated circuits, circuit modeling, intermodulation distortion, mixers.**

I. Introduction

THE double-balanced bipolar mixer [1] in Fig. 1 is commonly used in integrated receivers for mixing a local-oscillator (LO) and radio-frequency (RF) signal to an intermediate frequency (IF). Double-conversion receivers often require mixers with high third-order intercept point (IP3) [2] to meet the system IP3 specification. However, in zero and low IF architectures, the even-order distortion terms are also of particular concern [3]. For example, leakage of the LO signal to the RF input of the mixer will create LO self-mixing terms that cause a dc offset at the IF output. Likewise, second-order intermodulation terms can appear if two tones present at the RF input experience second-order distortion. If the switching stage of the mixer is not ideal, the resulting low frequency leaks through to the IF output, degrading the second-order intercept point (IP2) of the mixer.

Mismatch combinations and offsets that may be present in the double-balanced mixer of Fig. 1 will be described in Section II. A theoretical analysis of intermodulation products due to these mismatches and local oscillator mark-to-space errors is presented in Section III. Five different scenarios that cause even-order distortion terms are discussed in Section IV. Finally, guidelines for the maximum mismatch permissible to meet a given IP2 specification based on analysis and simulation results are developed in Section V.

II. Mixer Mismatches and Offsets

Several mismatch combinations are possible in the mixer shown in Fig. 1. First, the RF input transistors Q5 and Q6 could be mismatched. This mismatch can be caused by several factors, including temperature gradients, voltage drops in metal lines, and mismatches in emitter area, current gain, or parasitic emitter resistances. All of these factors can contribute to an effective offset voltage $V_{\rm os}$ on Q5 relative to Q6. Also in the RF input stage, the emitter resistors R_{E1} and R_{E2} can mismatch. An offset of $V_{\rm os}$ on Q5 relative to Q6 has the same effect on even-order intermodulation products as R_{E2} mismatching R_{E1}

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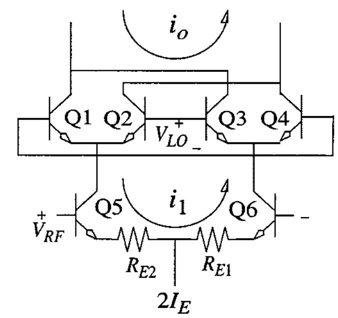

Fig. 1. Double-balanced mixer schematic.

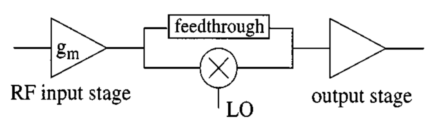

Fig. 2. Mixer model with input stage, mixing stage, output stage, and feedthrough path.

such that $I_E(R_{E1} - R_{E2}) = V_{\rm os}$. In the following sections, any offset of R_{E1} relative to R_{E2} will be included as an offset on Q5 relative to Q6 to simplify the analysis. The LO switching stage offsets generated from the same mechanisms mentioned above can also affect Q1–Q4. Three combinations of offsets are possible. These offsets are generated from the same mechanisms listed above. First, Q1 could have an offset from Q2 while Q3 and Q4 are matched. Secondly, both Q1 and Q3 could have the same offset relative to Q2 and Q4. Finally, an offset could appear on both Q1 and Q4. These offsets change the mark-to-space ratio of the local oscillator signal driving Q1 through Q4. If the LO signals have a nonideal mark-to-space ratio, that is, the duty cycle is not exactly 50%, the intermodulation products are affected in the same way as by corresponding offsets in Q1–Q4. The output of the mixer can drive load resistors, reactive loads such as inductors, or another gain stage. Mismatches can also occur in each of these stages.

III. Mixer Analysis

A typical integrated circuit double-balanced mixer can be modeled as shown in Fig. 2. The RF input stage is a linear transconductance amplifier. In the mixer shown in Fig. 1, the input stage consists of the transistors Q5 and Q6. The LO signal drives the switching stage that is implemented by Q1–Q4. Finally, the current output from the switching stage is applied to an output stage, which could consist of resistors, a reactive load, or a gain stage. The mixer model in Fig. 2 also shows a feedthrough path from the RF input stage to the IF output stage.

A. Effects of Mark-to-Space Errors on Spurious Signals

The LO signal is typically a square wave that has an on time referred to as the "mark" and an off time referred to as the "space." If the duty cycle is 50%, the mark-to-space ratio is ideal and equal to unity. If Q1–Q4 switch with a mark-to-space ratio exactly equal to unity, any even-order intermodulation terms generated in the RF input stage would be applied equally to the output nodes by the switching stage and cancel. Offset errors in the transistors can be combined with errors in the LO mark-to-space ratio to simplify the switching stage analysis. Each of the transistors Q1, Q2, Q3, and Q4 in Fig. 1 has an "on" time given by M_1, S_1, S_2, and M_2, respectively. In Fig. 1, the dc current through Q5 and Q6 at each collector is I_E. The variables i_o and i_1 are ac currents. The current in Q5 is then $I_E + i_1$ and the current in Q6 is $I_E - i_1$. Likewise, the current in the first output node is $I_E + i_o$ and the current in the second output node is $I_E - i_o$. The spurious output current i_o has a signal component and a dc component as shown in (1), where $M + S$ is the LO period and is equal to $M_1 + S_1$ and $M_2 + S_2$

$$i_o = \frac{M_1 - S_1 + M_2 - S_2}{2(M+S)} \cdot i_1 + \frac{M_1 - S_1 - (M_2 - S_2)}{2(M+S)} \cdot I_E. \tag{1}$$

The signal current from the RF input stage is i_1. The first term of (1) is the spurious signal component, and the second term is the dc component of the output current. If the mark-to-space ratio is unity, $M_1 = S_1 = S_2 = M_2$ and $i_o = 0$. No spurious ac signals at the RF frequency and no dc components appear at the output. Only the mixing components of the RF and LO frequency appear at the IF output. To simplify the analysis of (1), two cases will be considered. First, when $M_2 = M_1$ and therefore $S_2 = S_1$

$$i_o = \frac{M - S}{M + S} \cdot i_1. \tag{2}$$

This is the situation when a mark-to-space error exists in the LO signal. This case could also occur when both Q1 and Q4 have an offset in the same direction from Q2 and Q3. The output current i_o has no dc component of the tail current $2I_E$, but frequencies in the signal current i_1, including dc, can appear as part of i_o. The second case is where $S_2 = M_1$ and $S_1 = M_2$. This is caused by Q1 and Q3's both having an offset in the same direction compared to Q2 and Q4. Assuming that these offsets are positive, dc current flows in Q1 and Q3 for a longer period of time than Q2 and Q4. The currents in Q1 and Q3 are summed, and a dc offset appears at the output. This offset is given by

$$i_o = \frac{M_1 - S_1}{M + S} \cdot I_E = -\frac{M_2 - S_2}{M + S} \cdot I_E. \tag{3}$$

B. Effect of Mark-to-Space Errors on Conversion Gain

Errors in the LO drive mark-to-space ratio will lower the mixer conversion gain as well as contribute to second-order intermodulation products at the mixer output. With an LO signal that has a perfect mark-to-space ratio, the mixer will have a conversion gain from the RF input to the IF output of $2/\pi$ or -4 dB. As the mark-to-space ratio degrades, the gain will decrease.

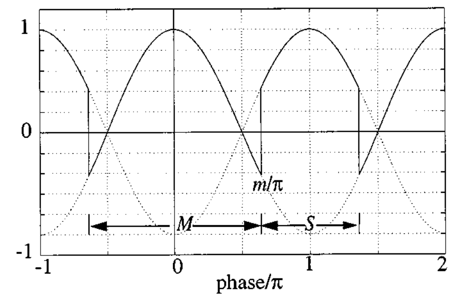

Fig. 3. Normalized mixer output currents with uneven mark-to-space ratio.

First, consider an input current to the switching stage of $i_1 = I_1 \cos(\omega_{\mathrm{RF}} t)$, where $\omega_{\mathrm{RF}} = 2\pi f_{\mathrm{RF}}$, and an LO signal with frequency $\omega_{\mathrm{LO}} = 2\pi f_{\mathrm{LO}}$. The LO signal and the RF signal will move in and out of phase with each other at the frequency of the IF, $f_{\mathrm{IF}} = f_{\mathrm{LO}} - f_{\mathrm{RF}}$. Assuming $f_{\mathrm{RF}} \gg f_{\mathrm{IF}}$, at the point in time when the LO and RF signals are in phase, the IF signal consists of the rectified RF signal. The average value of the rectified sine wave is $2/\pi$, which is the peak value of the IF signal and therefore the ideal gain of the mixer. If the LO mark-to-space ratio is not unity, this gain degrades, as shown in Fig. 3. The vertical axis in Fig. 3 is the output current normalized to unity. The equation for the gain degradation due to mark-to-space errors can be calculated by deriving the mixer gain as described above and including the mark-to-space variations. Taking the integral of the RF signal and evaluating as a function of m, where m is a function of phase angle and equals $\pi M/(M+S)$

$$\frac{i_o}{I_1} = \frac{\omega}{\pi}\left[\int_0^{m/\omega} \cos\omega t \, dt + \int_{m/\omega}^{\pi/\omega} (-\cos\omega t) \, dt\right] \tag{4}$$

$$= \frac{2}{\pi} \sin m. \tag{5}$$

Converting m from a function of phase angle to the mark-to-space ratio terms M and S, (5) becomes

$$\frac{i_o}{I_1} = \frac{2}{\pi} \sin\left(\frac{M\pi}{M+S}\right) \tag{6}$$

$$= \frac{2}{\pi} \sin\left(\frac{\pi}{2}\left(\frac{M-S}{M+S} + 1\right)\right). \tag{7}$$

Equation (7) shows that if the mark-to-space ratio is unity, $M = S$ and the gain of the switching stage is $2/\pi$ as expected. As the mark-to-space error increases, the gain degrades. Equation (7) is normalized to $2/\pi$ and plotted in Fig. 4.

C. Conversion Gain as a Function of Feedthrough

The mixer model shown in Fig. 2 can now be expanded to include the effects of mark-to-space errors on gain and second-order intermodulation products. This expanded model is shown in Fig. 5. The first gain stage models the RF input stage. The variables a_1 and a_2 represent the first- and second-order expansion coefficients, respectively. The variable e_1 is the small signal at the RF input, and e_2 is the small signal at the input to the switching stage. The model of the switching stage includes a path to represent the conversion loss of $2/\pi$ and a path

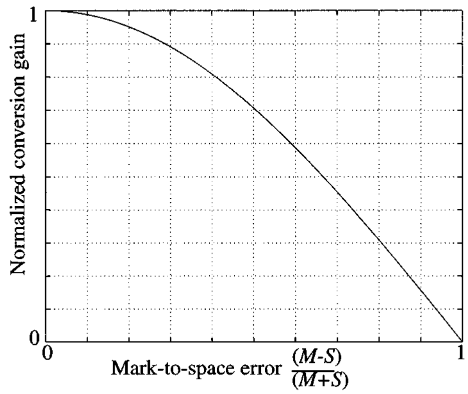

Fig. 4. Effect of LO mark-to-space error on conversion gain.

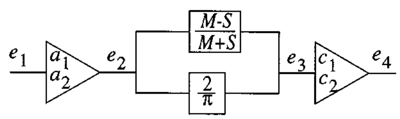

Fig. 5. Mixer with factors for feedthrough, conversion loss, and first- and second-order gain coefficients.

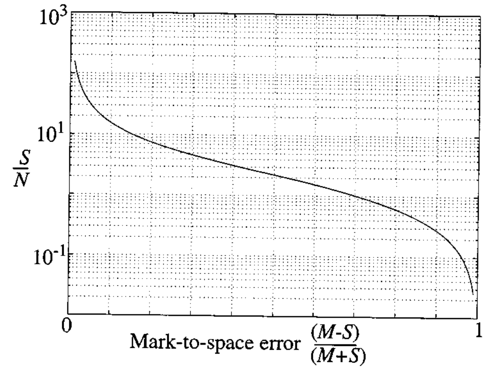

Fig. 6. Ratio of desired converted signal to undesired unconverted feedthrough noise as a function of LO mark-to-space error.

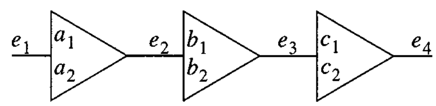

Fig. 7. Cascade of amplifiers with first- and second-order gain coefficients.

to show the feedthrough of signals from the input stage to the IF output. Finally, the output of the mixer is shown as a gain stage having first- and second-order expansion coefficients of c_1 and c_2, small signal input of e_3 and output of e_4. The two paths though the switching stage illustrate that both a signal present at the RF input and a signal at the IF frequency can appear at the output.

The ratio of the conversion gain of the wanted IF signal to the feedthrough gain of unwanted signals including RF and dc is expressed as a signal-to-noise ratio (S/N)

$$\frac{S}{N} = \frac{\frac{2}{\pi}\sin\left(\frac{M\pi}{M+S}\right)}{\frac{M-S}{M+S}}. \tag{8}$$

Equation (8) is plotted in Fig. 6. If the mark-to-space ratio is unity, $(M-S)/(M+S)=0$ and the S/N ratio from (8) goes to infinity. As the mark-to-space error degrades, the S/N ratio becomes less than infinity. In the limit of $(M-S)/(M+S)=1$, the S/N ratio becomes zero.

D. IP2 of Cascaded Amplifiers

So far, effects of LO mark-to-space errors on conversion gain and feedthough have been discussed. In the following section, effects of nonlinearities and offsets in the RF input stage and mixer output stage will be considered. To analyze the combined effects of nonidealities in each stage, the second-order intermodulation products of cascaded stages will be examined.

First, consider three cascaded amplifiers as shown in Fig. 7. The first amplifier has first- and second-order voltage gain coefficients a_1 and a_2. Similarly, the second amplifier has coefficients b_1 and b_2, and the third amplifier has coefficients c_1 and c_2. The signals e_1, e_2, e_3, and e_4 are the signals at each point in the amplifier cascade.

The transfer function for each amplifier is defined as

$$e_{n+1} = a_1 e_n + a_2 e_n^2 + \cdots. \tag{9}$$

With $n=1$, the transfer function of the first stage is

$$e_2 = a_1 e_1 + a_2 e_1^2 + \cdots. \tag{10}$$

The IP2 occurs when the first- and second-order terms are equal, i.e., when $a_1 e_1 = a_2 e_1^2$. Defining the intercept voltage of the nth stage as IV_n, the intercept voltage of the first amplifier is

$$\frac{1}{IV} = \frac{1}{e_1} = \frac{a_2}{a_1} = \frac{1}{IV_a}. \tag{11}$$

The transfer function for the second amplifier is

$$e_3 = b_1 e_2 + b_2 e_2^2 + \cdots. \tag{12}$$

Substituting e_2 from (10) into (12) and neglecting higher order terms, the transfer function from e_1 to e_3 is

$$e_3 = a_1 b_1 e_1 + (a_2 b_1 + a_1^2 b_2 + \cdots) e_1^2. \tag{13}$$

Equating the first- and second-order terms, the intercept for two stages occurs when

$$\frac{1}{IV} = \frac{1}{e_1} = \frac{a_2}{a_1} + a_1 \frac{b_2}{b_1} = \frac{1}{IV_a} + \frac{a_1}{IV_b}. \tag{14}$$

Using a similar analysis for three stages, the intercept point is given by

$$\frac{1}{IV} = \frac{1}{IV_a} + \frac{a_1}{IV_b} + \frac{a_1 b_1}{IV_c}. \tag{15}$$

Equation (15) shows that the second-order voltage intercept points can be cascaded in a manner similar to those of third-order power intercepts.

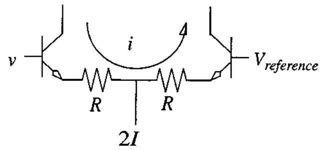

Fig. 8. Degenerated differential amplifier.

E. Differential Amplifier IM2 Caused by an Input Offset

The mixer RF input stage shown in Fig. 1 is a differential amplifier. The transfer function of an ideal differential amplifier with input signal e and input offset E is given by

$$e_o = a_1(e+E) + a_3(e+E)^3 + a_5(e+E)^5 + \cdots. \quad (16)$$

Expanding the $e+E$ terms assuming the offset E to be small, (16) can be simplified to

$$e_o = a_1E + a_1e + 3a_3Ee^2 + a_3e^3 + \cdots. \quad (17)$$

Equating first- and second-order terms in (17), the second-order intercept point occurs when

$$e = \frac{1}{3E} \cdot \frac{a_1}{a_3}. \quad (18)$$

Equations (16)–(18) apply to an ideal differential gain stage. Degeneration is often used to get a more linear RF input stage. The degenerated differential amplifier in Fig. 8 has the transfer function

$$\frac{v}{2V_T} = \tanh^{-1}\left(\frac{i}{I}\right) + \frac{2Ri}{2V_T} \quad (19)$$

where $V_T = kT/q$ is the thermal voltage and v is the ac input voltage. Expanding the $\tanh^{-1}$ function and collecting terms gives

$$\frac{v}{2V_T} = \left(1+\frac{RI}{V_T}\right)\frac{i}{I} + \frac{1}{3}\left(\frac{i}{I}\right)^3 + \frac{1}{5}\left(\frac{i}{I}\right)^5 + \cdots. \quad (20)$$

To solve for the relationship of i in terms of I, the power series in (20) is reversed [4] with the result

$$\frac{i}{I} = \frac{1}{1+\frac{RI}{V_T}} \cdot \frac{v}{2V_T} - \frac{1}{3} \cdot \frac{1}{\left(1+\frac{RI}{V_T}\right)^4} \cdot \left(\frac{v}{2V_T}\right)^3 + \cdots. \quad (21)$$

Introducing a dc offset E to the RF input of the mixer and substituting the first- and third-order coefficients from (21) into (18), the input intercept point referenced to a load impedance of R_L will occur when

$$\frac{v}{2V_T} = \frac{1}{3 \cdot \frac{E}{2V_T}} \cdot 3\left(1+\frac{RI}{V_T}\right)^3. \quad (22)$$

In this case, the coefficients a_1 and a_3 were normalized to the form of $v/2V_T$ and $E/2V_T$, respectively. The intercept point, however, evaluates to the same value regardless of the normalization form. Solving (22) for v

$$v = \frac{(2V_T)^2}{E}\left(1+\frac{RI}{V_T}\right)^3. \quad (23)$$

Converting from a second-order voltage intercept point to a second-order power intercept point (IIP2) in dBm referenced to an impedance R_L gives

$$\text{IIP2} = 10\log\left(\frac{(2V_T)^2}{2R_L}\right) + 30 + 60\log\left(1+\frac{RI}{V_T}\right) - 20\log\left(\frac{E}{2V_T}\right) \text{ dBm}. \quad (24)$$

The final term in (24) shows a 6-dB reduction of the intercept point for each $2\times$ increase in the offset voltage. Therefore, increasing the amount of emitter degeneration will improve the IIP2 by reducing the impact of the offset of the differential pair. However, this will decrease the input stage gain for a given load resistance, which will reduce the improvement in the output IP2.

The analysis thus far has concentrated on the effect of offset on a differential amplifier stage. If this stage is now used as the RF input stage to a mixer, a complete analysis of the overall IP2 must include degradation due to conversion loss of the first-order product and any second-order products that appear at the IF output as a result of feedthrough. For this case, (24) is modified as follows:

$$\text{IIP2} = 10\log\left(\frac{(2V_T)^2}{2R_L}\right) + 30 + 60\log\left(1+\frac{RI}{V_T}\right) - 20\log\left(\frac{E}{2V_T}\right) - 20\log\left(\frac{M-S}{M+S}\right) + 20\log\left(\frac{2}{\pi}\sin\left(\frac{\pi}{2}\left(\frac{M-S}{M+S}+1\right)\right)\right) \text{ dBm}. \quad (25)$$

For small mark-to-space errors, the last term in (25), which represents the conversion loss, is approximately $20\cdot\log(2/\pi) = 4$ dB. This term can be combined with the other constants in the first term.

IV. Causes of Even-Order Distortion

The previous section developed a generic equation for mixer IP2 in terms of offsets and LO mark-to-space ratios. More specific cases where even-order intermodulation is generated and how (25) can be applied will be examined in this section.

A. Low-Frequency Feedthrough

If a low-frequency signal is present at the RF input, an offset in the switching stage will allow this signal to be fed through to the mixer output as shown in Fig. 9. The spurious signal level at the IF output depends only on the switching transistor mismatch. Mismatch in the RF input stage has no effect on the magnitude of the low frequency seen at the output. The gain of the low-frequency signal from the input to the output is given by

$$A_{LF} = a_1c_1\frac{M-S}{M+S} \quad (26)$$

where M and S are the switching stage mark-to-space ratio, a_1 is the RF input stage gain, and c_1 is the gain of the mixer output stage.

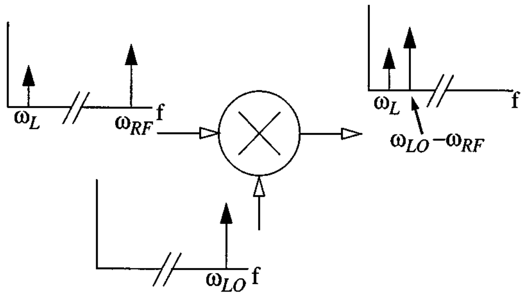

Fig. 9. Low-frequency feedthrough due to offsets in the switching transistors.

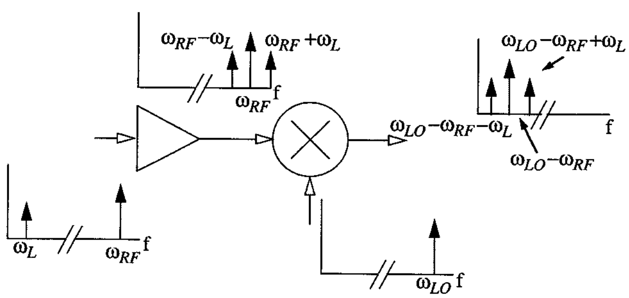

Fig. 10. RF and low-frequency intermodulation.

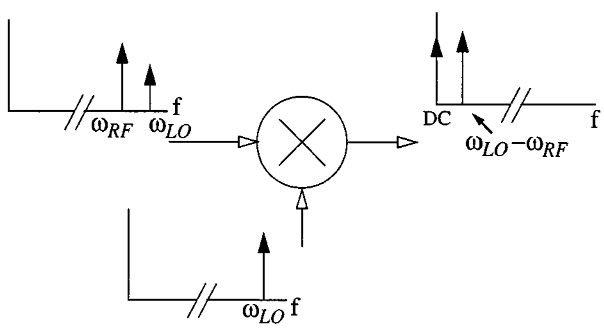

Fig. 11. Local oscillator self-mixing.

B. RF and Low-Frequency Intermodulation

A low-frequency signal at frequency ω_L at the RF input will mix with the RF signal at frequency $\omega_{\rm RF}$ in the presence of second-order distortion and will create second RF tones at $\omega_{\rm RF}-\omega_L$ and $\omega_{\rm RF}+\omega_L$. Both RF tones will mix with the local oscillator signal, generating the desired IF signal at $\omega_{\rm LO}-\omega_{\rm RF}$ and the interferers at $\omega_{\rm LO}-\omega_{\rm RF}+\omega_L$ and $\omega_{\rm LO}-\omega_{\rm RF}-\omega_L$, as shown in Fig. 10. In this situation, only offsets in the RF input stage Q5 and Q6 and the emitter resistors affect the magnitude of the interferers. The second-order input intercept point is simply that of the input stage IP2_a shown in (24).

C. Local Oscillator Self-Mixing

Leakage from the LO input to RF input creates intermodulation as shown in Fig. 11. The LO signal on the RF port is mixed with the LO signal, causing both the IF signal and a dc component to appear at the output. This degrades the performance of zero IF receivers.

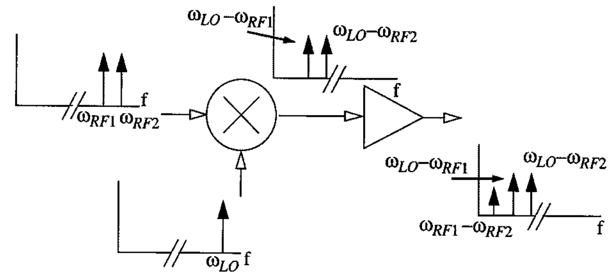

Fig. 12. Output nonlinearity.

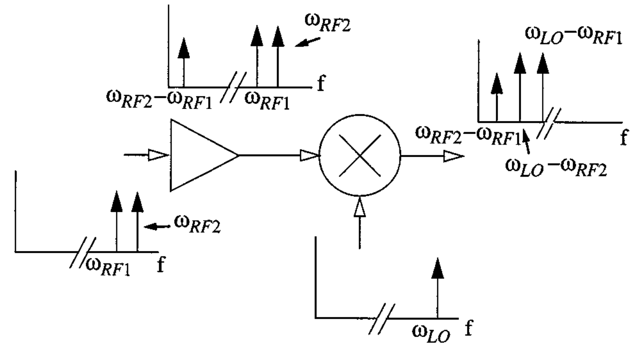

Fig. 13. Nonlinearity in the RF and LO stages.

D. Output Nonlinearity

Two tones at the RF input of the mixer at frequencies $\omega_{\rm RF1}$ and $\omega_{\rm RF2}$ will mix with the local oscillator signal creating frequencies $\omega_{\rm LO}-\omega_{\rm RF1}$ and $\omega_{\rm LO}-\omega_{\rm RF2}$ at the mixer output. If nonlinearities exist in the following stage or in the mixer load, the two difference frequencies can mix to form a spur at $\omega_{\rm RF1}-\omega_{\rm RF2}$, as shown in Fig. 12, even though no second-order intermodulation products were observed at the mixer output.

E. Nonlinearity in the RF and LO Stages

When two tones at $\omega_{\rm RF1}$ and $\omega_{\rm RF2}$ are present at the RF input of the mixer, the sum and difference frequencies can be generated as a result of offsets in the RF input stage. If the switching transistors have an offset, then the difference frequency $\omega_{\rm RF1}-\omega_{\rm RF2}$ can then leak through to the following stage. The IIP2 in this situation, shown in Fig. 13, is described by (25). From Fig. 5, the signals at the output of each stage are given by the following three equations:

$$e_2 = a_1 e_1 + a_2 e_1^2 + \cdots \tag{27}$$

$$e_3 = \frac{2}{\pi} a_1 e_1 + \left(\frac{M-S}{M+S}\right) a_2 e_1^2 + \cdots \tag{28}$$

$$e_4 = c_1 e_3 + c_2 e_3^2 \cdots . \tag{29}$$

The mark-to-space error is assumed to be small enough to have negligible effects on the conversion gain. Solving for e_4 in terms of e_1, simplifying and neglecting higher order terms

$$e_4 = \frac{2}{\pi} a_1 c_1 e_1 + \left(\frac{M-S}{M+S} a_2 c_1 + \left(\frac{2}{\pi}\right)^2 a_1^2 c_2 + \cdots\right) e_1^2. \tag{30}$$

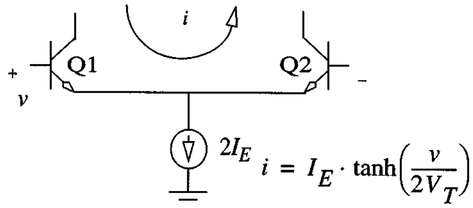

Fig. 14. Schematic of a differential stage used to model the LO switching circuit in Matlab.

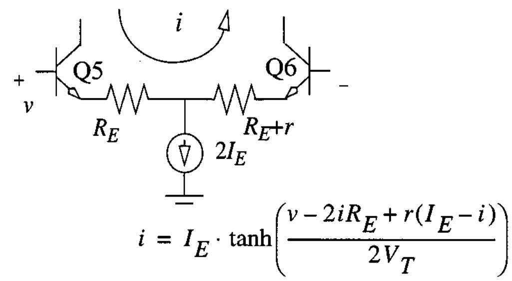

Fig. 15. Schematic of a degenerated differential stage used to model the RF input circuit.

Equating first- and second-order terms gives the voltage at the intercept

$$\frac{1}{e_1} = \frac{\pi}{2} \cdot \frac{M-S}{M+S} \cdot \frac{a_2}{a_1} + \frac{2}{\pi} \frac{a_1 c_2}{c_1}. \tag{31}$$

In terms of intercept points of each stage

$$\frac{1}{IV} = \frac{\pi}{2} \cdot \frac{M-S}{M+S} \cdot \frac{1}{IV_a} + \frac{2}{\pi} \cdot \frac{a_1}{IV_c}. \tag{32}$$

The mixer intercept voltage IV is a function of the intercept voltage IV_a and gain a_1 of the RF input stage, the conversion loss of the mixer, the mark-to-space error from the switching stage, and the intercept voltage of the output stage IV_c. Equation (32) assumes a small mark-to-space error in the LO signal. For large mark-to-space error, the assumption is no longer valid and the first $\pi/2$ term would have to be replaced by $((2/\pi)\sin(M\pi/M+S))^{-1}$ from (6) to represent the mixer conversion loss. Equation (32) does not include the effect of a dc error fed from the output of the switching stage into the output stage of the mixer. This dc error would decrease the intercept point of the output stage IV_c. This error can feed through the mixer from the input stage or be created by offsets in the switching stage of the mixer.

Five situations were examined that cause even-order intermodulation products. Intermodulation products can be reduced by matching the switching transistors Q1–Q4 and the RF input transistors Q5 and Q6 from Fig. 1. Also, limiting the low frequencies that appear at the RF input and improving LO to RF isolation will reduce spurious signals at the IF.

V. Analytical Results

The double-balanced mixer was modeled in Matlab, and the results were compared to SPICE simulations. The SPICE simulation used a bipolar transistor model with $f_\tau = 20$ GHz. In Matlab, the two differential pairs used in the LO switching stage were modeled as shown in Fig. 14. The local oscillator drive signal was a sum of sine waves to generate a square wave with a variable mark-to-space ratio so that effects of mismatch could be simulated. The model shown in Fig. 15 was used in Matlab to study the effects of the emitter resistor variation in the RF input stage. An offset on the RF input differential pair was modeled by an ideal voltage source on the base of Q5. Good correlation was found between Matlab and SPICE results up to 500 MHz. Above 500 MHz, the IP2 obtained from the SPICE simulation degraded from the theoretical value obtained from Matlab due to parasitics, coupling, and lower transistor gain. The frequency where this correlation degraded was process dependent.

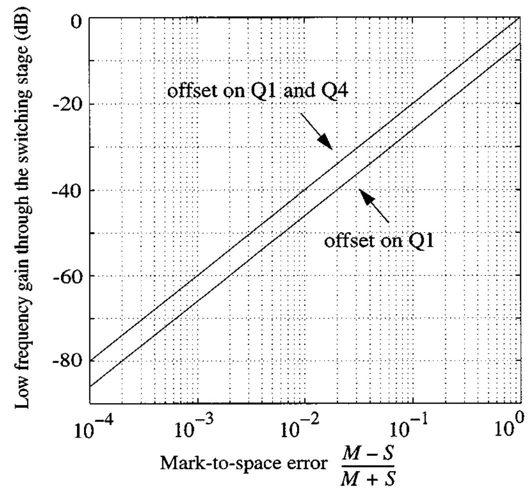

Fig. 16. Low-frequency feedthrough at the IF output for offsets on Q1 and on Q1 and Q4.

A. Low-Frequency Feedthrough

To simulate feedthrough, a low-frequency signal of frequency ω_L was applied to the RF input. First, the mark-to-space ratio of the signals driving the switching transistors was varied to simulate an offset on Q1. Next, a simulation was performed with an equal offset applied to Q1 and Q4. For these two simulations, the magnitude of ω_L at the IF output was measured for varying mark-to-space ratios. Finally, an offset on Q1 and Q3 was simulated. With this offset combination, the signal feedthrough at ω_L at the IF output was canceled and a dc offset was generated that increased as offsets in LO signal increased. The magnitude of the dc offset is dependent on I_E. The gain through the switching stage of the low frequency signal at ω_L is plotted in Fig. 16. These simulation results correspond to (1). An offset on only Q1 causes 6 dB less feedthrough than an offset on Q1 and Q4. Better matching of Q1–Q4 will decrease the spurious signals appearing at the IF output.

B. RF Input Stage IP2

To simulate the input IP2 of a degenerated differential amplifier in Matlab, the RF input signal was modeled as the sum of two sine waves at frequencies ω_{RF1} and ω_{RF2}. Initially, both emitter resistors were zero. An offset was applied to Q5, and the IP2 was measured. Next, the IP2 was simulated for a dc voltage drop across the degeneration resistors equal to the thermal voltage. The resulting IIP2 is plotted in Fig. 17 as a function

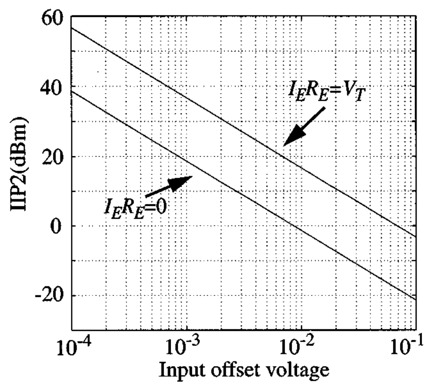

Fig. 17. IIP2 of a degenerated differential amplifier for two values of emitter resistors.

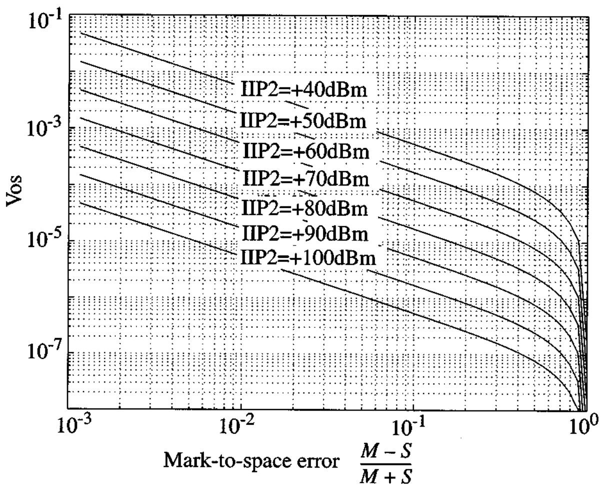

Fig. 18. Lines of constant IIP2 for an offset voltage on Q5, LO mark-to-space errors, and $R_E = 0$.

of offset voltage. These simulation results correspond to (24). Increasing the linearization in the RF input stage decreases the effects of mismatch. Therefore, as the emitter resistor is increased, the mismatch can be larger while still minimizing the undesired intermodulation. Equation (24) predicts that the IIP2 of a differential amplifier is very sensitive to emitter resistance. If the amplifier stage does not have degeneration resistors, the parastic emitter resistance and any contact resistance may also need to be considered to get accurate results.

C. Nonlinearity in the RF and LO Stages

To simulate the effects of nonlinearities in both the RF and LO stages, the input signal was modeled as the sum of two sine waves at frequencies ω_{RF1} and ω_{RF2}. Both emitter resistors were set to $R_E = 0\ \Omega$. The mark-to-space ratio of the voltage signals driving the switching transistors was varied so an effective offset appeared on Q1 and Q4. The RF input offset V_{os} on Q5 was varied from 0 to 10 mV. The resulting lines of constant input IIP2 are plotted in Fig. 18 as a function of V_{os} and the LO mark-to-space ratio with $R_E = 0$. The simulated data correspond with (25). As R_E increases, the lines of constant IIP2 will shift upwards. If an IIP2 of +50 dBm is required for $R_E = 0$, and the offset voltage on Q5 is known to be as

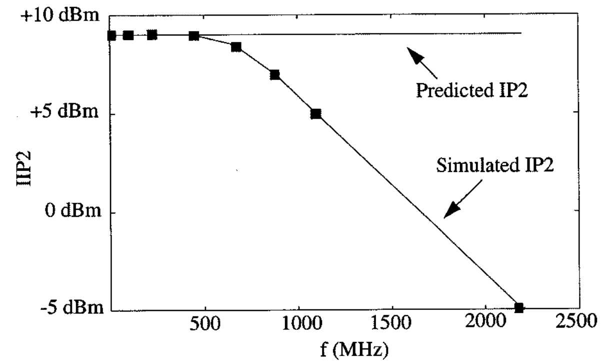

Fig. 19. Comparison of results calculated with (25) to results simulated in SPICE using a bipolar transistor model with $f_\tau = 20$ GHz.

large as 2 mV, the maximum allowable mark-to-space variation at low frequencies is $(M - S)/(M + S) < 9 \cdot 10^{-3}$. This corresponds to a duty cycle of 50.45%. At higher frequencies, the IIP2 will degrade and better matching will be required to meet the IIP2 specification. The duty cycle constraint can be relaxed by increasing R_E at the expense of lower conversion gain.

A typical comparison between IP2 obtained with SPICE simulations and calculated with (25) is shown in Fig. 19. For this particular process, good correlation is obtained up to 500 MHz. By 900 MHz, there is a 3-dB discrepancy. The IP2 obtained from SPICE degrades due to parasitic capacitance, coupling, and lower transistor gain.

Finally, if the RF input stage is not a simple degenerated differential pair, the total mixer IIP2 can be calculated from (25) given the IIP2 of the mixer input stage (IIP2_{in}) and the mark-to-space ratio of the switching transistors as shown

$$\text{IIP2} = \text{IIP2}_{\text{in}} - 20\log\left(\frac{M-S}{M+S}\right) + 20\log\left(\frac{2}{\pi}\sin\left(\frac{\pi}{2}\left(\frac{M-S}{M+S}+1\right)\right)\right). \quad (33)$$

VI. Conclusions

Several conclusions can be drawn from this study. First, even-order intermodulation products can be generated by several methods, including low-frequency feedthrough, RF and low-frequency intermodulation, LO self-mixing, nonlinearities in the output, and nonlinearities in both the LO and RF stages. Some of these even-order intermodulation terms, such as those caused by LO to RF leakage, can be reduced by improving isolation. Also, the mixer output stage and following stage must also have good IP2 so that the overall system IP2 is acceptable. Finally, the maximum achievable IP2 for two RF tones at the mixer input can be expressed in terms of offsets in the RF input stage and LO duty cycle errors. However, at higher frequencies, the actual IP2 will be less than the predicted value due to parasitics, coupling, and lower transistor gain.

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REFERENCES

[1] A. Bilotti, "Applications of a monolithic analog multiplier," *IEEE J. Solid-State Circuits*, vol. SC-3, pp. 373–380, Dec. 1968.

[2] R. C. Meyer, "Intermodulation in high-frequency bipolar transistor integrated-circuit mixers," *IEEE J. Solid-State Circuits*, vol. SC-21, pp. 534–537, Aug. 1986.

[3] B. Razavi, "Design considerations for direct conversion receiver," *IEEE Trans. Circuits Syst. II*, vol. 44, pp. 428–435, June 1997.

[4] M. R. Spiegel, *Mathematical Handbook of Formulas and Tables*. New York: McGraw-Hill, 1968.

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